Combined Integrity of GPS and Galileo



Integrity as the measure of trust placed in the correctness of the information provided by navigation systems is clearly one key factor of safety critical applications, such as precision landing procedures and precise maritime harbor applications. With the existing protection level concept employed by SBAS + GPS and the computation of the integrity risk at the alarm limit used within the upcoming Galileo baseline integrity concept, two different approaches will be available within the near future. Although both concepts share the same objective to describe the integrity of a user – and therefore have several basics in common – each respective system's usage of information provided by the other concept is not foreseen. This column examines GPS and Galileo integrity methods and proposes an algorithm for a combined integrity approach using data from both, as well as practical issues of implementation and computation of associated protection levels.

FELIX KNEISSL AND CARSTEN STÖBER UNIVERSITY FAF MUNICH ntegrity denotes the measure of trust placed in the correctness of the information provided by navigation systems. Safety critical applications require integrity measures to indicate with what level of confidence the navigation information may be used.

Although the existing protection level concept employed by satellite-based augmentation system (SBAS) combined with GPS provides one method to estimate this level of trust, the upcoming Galileo system will employ a different approach — computation of the integrity risk at the alarm limit. Thus, two different approaches will be available within the near future. However, neither the SBAS + GPS method nor the Galileo plan makes use of any additional information provided by other integrity data sources. This column presents an algorithm and demonstration of a combined integrity approach using data from both integrity concepts. The algorithm is based on the integrity data available on the user side. The presentation will also explore practical implementation issues and the computation of protection levels for calculated integrity risks.

Visions of Integrity

Safety critical applications require consideration of the measure of trust for the position solution derived from the navigation solution. This measure of trust is known as integrity. Users may determine their integrity by receiver autonomous algorithms (RAIM), by using external integrity data sources such as SBAS, or by using integrity data provided within the navigation message as it will be provided by Galileo. This column is dedicated to considerations of the last two approaches, while it has to be pointed out that RAIM algorithms will be part of an overall integrity solution in all cases.

Currently, the SBAS + GPS integrity concept is the only existing source of integrity information for GNSS users



on a regional basis. In the form of the Wide Area Augmentation System (WAAS) this concept has been successfully implemented in North America and will be available in Europe through the European Geostationary Navigation Overlay System (EGNOS) within the near future.

A second approach to provide integrity information is foreseen within the current Galileo baseline integrity concept, which is intended to work on a global basis. Both concepts provide different information and use different methods to calculate the integrity measure. The SBAS + GPS approach provides both a *horizontal protection level* (HPL) and a vertical protection level (VPL), while the Galileo approach calculates the overall *integrity risk*, P_{HMI} .

Nevertheless, neither the SBAS + GPS integrity concept nor the Galileo integrity plan takes into account the integrity information provided by the other system. This has two consequences: Not incorporating additional information from the other system reduces the complexity of the integrity algorithms, and, as a result, simplifies the certification process because only the system itself has to be certified.

The main disadvantage of a "single system integrity algorithm" is that all modern GNSSs share most of their working principles and provide mostly equivalent measurement data, while at the same time suffering from similar propagation and measurement errors. As a result, the transmitted information needed for integrity assessment is comparable, similar, or even identical.

This system commonality does not justify neglect the additional data even if they are provided by different integrity sources, because the more measurements and integrity information are utilized the better the knowledge of the derived user integrity will be. The only remaining restriction is that the additional integrity source has to be trustworthy and certified.

In this column, after reviewing both integrity concepts in terms of similarities, differences, and basic definitions, we will present combined algorithms using simultaneous data from the SBAS + GPS and the Galileo integrity concept. As background to our discussion of these algorithms, the sidebar "User Integrity Concepts" (begins on page 54) discusses several key properties of the SBAS + GPS and Galileo integrity concepts.

In general both integrity concepts share basic principles such as signal-inspace (SIS) error and user-to-satellite geometry. Moreover, the fault-free allocation tree within the Galileo integrity concept is very similar to the SBAS + GPS integrity concept, except for (a) the representation of the final result, and (b) the non-fixed allocation for the different protection domains in Galileo.

In order to provide an adequate comparison, we describe the outcome of the SBAS + GPS algorithm in terms of integrity risks (IR) at the alarm limit (AL) and the outcome of the baseline Galileo integrity concept is described in terms of protection levels (PL) at given integrity risks. Our analysis will demonstrate that protection levels and integrity risks at the alert limit are mathematically an inversion of the same concept but cannot be compared directly. Particularly, different allocations of horizontal and vertical integrity risks allow the user to obtain a family of inverse points to the given horizontal and vertical protection levels.

To facilitate the comparison between SBAS+GPS and Galileo integrity concepts, a numerical implementation is presented that transforms integrity risks into protection levels and vice versa.

In addition to the combined integrity algorithms, the current baseline for the Galileo integrity concept was reviewed carefully and also implemented standalone. An outcome of the conducted simulations shows that, up to now, a major point missing in the Galileo integrity concept is the sustained consideration of non-signal-in-space (SIS) errors. Within publicly available studies only signal-in-space accuracy (SISA) and signal-in-space monitoring accuracy (SISMA) have been simulated and used to derive the theoretical performance of the Galileo integrity concept.

The column presents the integrity risk calculation regarding non-SIS failure sources because these error components are part of the final user integrity risk as well. The results of the tests carried out in this scope suggest that the fulfilment of the required system performance of the current Galileo baseline integrity concept will be challenging for the maximum allowed SISA, SISMA, and local error contributions.

Direct and Indirect Integrity Formulations

Within the following sections, the computation of integrity risks for a given user geometry and specific alarm limits will be denoted as the direct problem, whereas the computation of protection levels for a given user geometry and specific integrity risks will be denoted as the indirect problem.

The GPS + SBAS and Galileo integrity formulations are complementary. GPS + SBAS results in protection levels to given integrity risks, while the Galileo integrity formulation results in integrity risks for given alarm limits. The HPL specifies the maximum allowable horizontal deviation for which the *a priori* Working Papers continued on page 56

User Integrity Concepts

The following review only covers aspects of the SBAS + GPS and Galileo integrity concepts relevant to this column. Moreover, only data available on the user side is taken into account.

SBAS + GPS Integrity

The SBAS + GPS integrity concept is based on the broadcast of differential GPS corrections and corresponding integrity data transmitted by geostationary satellites. The essential input quantities for the integrity algorithm on the user side are:

- geometry between GPS satellites and user derived from observations of the GPS satellites
- user differential range error $\sigma_{_{UDRE}}$, transmitted by SBAS satellite
- grid ionospheric vertical error $\sigma_{_{\rm GIVE}}$, transmitted by the SBAS satellite
- tropospheric error σ_{tropo} derived from the model defined within the Radio Technical Commission For Aeronautics (RTCA) publication, *Minimum Operational Performance Standards (MOPS) For Global Positioning System/Wide Area Augmentation System Airborne Equipment*, RTCA DO-229D
- error of airborne receiver errors σ_{air}, calculated depending on receiver properties and models defined within RTCA DO-229D.

The final assessment of the user integrity calculates the HPL and VPL as depicted in (1) and (2):

$$HPL = \begin{cases} K_{H,NPA} \cdot d_{major} en route \text{ inclusive non-precision approach} \\ K_{H,PA} \cdot d_{major} \text{ precision approach mode} \end{cases}$$
(1)

 $VPL = K_{V, PA} \cdot d_U$

The inflating factors K are based on fixed confidence intervals and the semi-major axis of the error ellipse d_{major} is calculated as:

$$d_{major} = \sqrt{\frac{d_{east}^2 + d_{north}^2}{2}} + \sqrt{\left(\frac{d_{east}^2 - d_{north}^2}{2}\right)^2 + d_{EN}^2}$$
(3)

with

$$\begin{bmatrix} d_{east}^2 & d_{EN} & d_{EU} & d_{ET} \\ d_{EN} & d_{north}^2 & d_{NU} & d_{NT} \\ d_{EU} & d_{NU} & d_U^2 & d_{UT} \\ d_{ET} & d_{NT} & d_{UT} & d_T^2 \end{bmatrix} = (\mathbf{G}^T \mathbf{W} \mathbf{G})^{-1}$$
(4)

The ith row of the geometry matrix G is defined based on the elevation *El* and the azimuth Az of the ith observed satellite as

$$\mathbf{G}_{i} = \left[-\cos El_{i} \sin Az_{i} - \cos El_{i} \cos Az_{i} - \sin El_{i} 1\right]$$
(5)

The weight matrix *W* is modelled in this context under assumption of uncorrelated (corrected) measurements characterized by the variance σ_i^2 for the ith observed satellite, as follows:

$$\sigma_i^2 = \sigma_{i,flt}^2 + \sigma_{i,UIRE}^2 + \sigma_{i,air}^2 + \sigma_{i,tropo}^2$$

Within (6) $\sigma_{_{UIRE}}$ for the user position is derived from $\sigma_{_{GIVE}}$ and $\sigma_{_{ff}}$ is derived from $\sigma_{_{UDRE}}$ including degradation parameters.

Key assumptions within the SBAS + GPS integrity concept include:

- The SBAS ground segment which generates the SBAS data is capable of identifying hazardous events related to the user equation part of the integrity risk allocation tree at all times. This implies per definition that all satellites indicated as healthy by the SBAS data are/have to be considered healthy by the user, without any knowledge of the accuracy of the monitoring process at the SBAS ground segment.
- The second assumption is that all errors utilized within (6) provide conservative estimations of the single error components resulting in a conservative estimation of the error variance σ_i.

In early versions of the RTCA DO-229 for augmented GNSS, the rationale of the K-factors was given in the assumption that an overbounded range error results in an overbounded position error. Furthermore, for en route operations the horizontal position error is assumed to be conservatively described by the semi-major axis of the error ellipse together with the Rayleigh distribution.

As it was not possible to assure bias-free and unimodal range error distributions allowing to apply cumulative distribution function (cdf) overbounding for vertical errors, the present version of the RTCA DO-229 for augmented GNSS states that the choice of the K-factors is "somewhat arbitrary."

The corrections and integrity information are provided by the SBAS system in order to guarantee that the protection levels computed using the specified K-factors hold valid for the given integrity risks, making it impossible to utilize the SBAS information in other applications without detailed knowledge of the ground segment algorithms. To be able to estimate the capabilities of a combined system all the same, the rationales given in an earlier iteration of the GPS/WAAS MOPS (RTCA DO-229C) have to be assumed.

Galileo Baseline Integrity Concept

The current baseline for the Galileo integrity concept, which we present to the best of our knowledge here, has been published in the papers by by V. Oehler et alia (2005, 2006) listed in the Additional Resources section near the end of this article. However, as far as we know, neither an official publication nor any official agreement exists on the final Galileo integrity concept.

The Galileo system design calls for the satellites themselves to broadcast the integrity information alongside the ranging data, resulting in the global availability of integrity data. Every user of the freely accessible Galileo Safety of Life Service will thus be able to assess the integrity of the navigation solution.

The essential input quantities for the integrity algorithm on the user side are:

- geometry between Galileo satellites and user position derived from observations of the Galileo satellites
- SISA as prediction of the expected SIS error, transmitted by the Galileo satellites

(6)

(2)

- SISMA comprising the accuracy of the monitoring process of the SIS error at the Galileo ground segment, transmitted by the Galileo satellites
- integrity flag transmitted by the Galileo satellites
- *horizontal alarm limit* (HAL) and *vertical alarm limit* (VAL), chosen by the user according to the designated application (e.g. landing approach)
- remaining errors.

The final assessment of the user integrity is done by calculation of P_{HMI} , the probability of hazardous misleading information. P_{HMI} is calculated as depicted in (7) depending on the HAL and VAL values, which are chosen by the user.

In contrast to the SBAS + GPS integrity concept, the Galileo integrity concept does not assume that all satellites indicating nominal health conditions are actually working nominally. Therefore, the overall integrity risk, $P_{\rm HMI}$, is calculated as the sum consisting of a "Fault-Free" (FF) and a "Faulty Mode" (FM) allocation tree.

$$P_{HMI}(VAL, HAL) = P_{IR,V} + P_{IR,H}$$

$$P_{HMI}(VAL, HAL) = P_{IR,V, FaultFreeMode} + P_{IR,V, FaultyMode}$$

$$+ P_{IR,H, FaultFreeMode} + P_{IR,H, FaultyMode}$$
(7)

 P_{HMI} consists of four independent parts derived from the combination of fault-free and faulty mode with horizontal and vertical integrity risks. Without further explanation equation (7) transforms to (8), where details can be found in [6].

$$\begin{split} P_{HMI}\left(VAL, HAL\right) &= \\ 1 - erf\left(\frac{VAL}{\sqrt{2} \cdot \sigma_{u,V,FF}}\right) + e^{\frac{HAL^2}{2\xi_{FF}^2}} \\ &+ \frac{1}{2} \cdot \sum_{j=1}^{N} P_{fail,sat_j} \cdot \left(\left(1 - erf\left(\frac{VAL + \mu_{u,V}}{\sqrt{2} \cdot \sigma_{u,V,FM}}\right)\right) + \left(1 - erf\left(\frac{VAL - \mu_{u,V}}{\sqrt{2} \cdot \sigma_{u,V,FM}}\right)\right)\right) \\ &+ \sum_{j=1}^{N} \left(P_{fail,sat_j} \cdot \left(1 - \chi^2_{2,\delta_{u,H}} cdf\left(\frac{HAL^2}{\xi_{FM}^2}\right)\right)\right) \end{split}$$

The *erf* (Gaussian error function) used within (8) identifies the integration of a probability density function (PDF) resulting in a cumulative distribution function (CDF). The last term of (8) contains a CDF of a non-central chi-squared distribution. For the calculation of the vertical integrity risk component of the fault free allocation tree the deviation $\sigma_{u,V,FF}$ is needed and is given by (10). The satellite to user geometry is contained within the topocentric projection matrix M_{topo} , where H is the design matrix, P is the weighting matrix and N_{topo} contains the orientation of the topocentric coordinate system.

$$M_{topo} = N_{topo}^{T} \cdot \left(H^{T} P H\right)^{-1} H^{T} P \tag{9}$$

$$\sigma_{u,V,FF}^{2} = \sum_{i=1}^{N} M_{topo} [3, i]^{2} \cdot \left(SISA_{i}^{2} + \sigma_{u,L,i}^{2} \right)$$
(10)

Equation (10) has been exemplarily chosen to point out $\sigma_{u,L,i}$, where *i* is an index for the ith Galileo satellite. The parameter

 $\sigma_{u,L,i}$ is used to identify the last point of the input quantities –the so called "remaining errors" – from the start of this paragraph and is of general interest.

According to the discussion in the paper by V. Oehler et alia (2005), all non SIS errors have to be handled by the user. This requires that all of these errors and variances —such as tropospheric error, receiver hardware delay, multipath and ionospheric error — have to be taken care of by the receiver manufacturers. To the knowledge of the authors, no official definition of the appropriate measures and procedures to do that has been offered up to now.

The final remaining error $\sigma_{u,L,i}$ in (10) is required to be a conservative estimation of the true error. Thus, receiver calibration and standardization will be necessary to ensure the integrity guarantees provided by the Galileo integrity concept.

Conclusions

Both integrity concepts specify the precision of the observables by providing conservative estimations of their standard deviation. The accuracy of the single-error components (e.g, orbit and clock error, ionospheric and tropospheric delay) is either sent via integrity messages or given by fixed computation orders. The known observation geometry is used to propagate measurement errors to the position domain.

The SBAS + GPS integrity concept defines that all GPS satellites which are considered healthy by the SBAS ground segment are working nominally and may be used by the user.

This comes from the fact that the integrity risk contributed from undetected faulty satellites is allocated in another branch of the allocation tree. This is equivalent to disregarding the faulty mode allocation tree used within the Galileo concept.

Both integrity concepts use vertical and horizontal components to assess the measure of integrity. Even within the Galileo integrity concept, the horizontal and vertical components are calculated indepen-

dently and summed up only within the last step. However, SBAS+GPS uses *fixed allocations* for the confidence intervals, while the Galileo concept uses *variable allocations*.

For the SBAS enlarging the 1- sigma confidence interval with the given K-factors results in a fixed split of the allocated integrity risk in a horizontal and vertical integrity risk associated with the computed protection levels of circa $1.0 e^{-7}$ and $2.0 e^{-8}$ ($1.0 e^{-7}$ is allocated for the "faulty mode"). For Galileo, the integrity risk is clearly dependent on the specified (and variable) alarm limit of interest.

SBAS+GPS characterizes the SIS error by $\sigma_{\rm fit}$, and Galileo uses SISA. Both quantities are derived from distributions assumed to be overbounded, which results in a conservative estimate of the true but unknown SIS error. The background of the variance σ_i^2 calculated via (6) is the same as that of (SISA_i²+ σ_{u,L_i}^2) as used within (10). Both quantities are comprised of the assumed SIS error plus all (modelled) remaining

errors and are used to characterize the accuracy of the ranging observation, which leads to the analogy expressed in equation (11).

Note, however, that (11) holds true for the physical meaning only, and not for the actual numerical values, because the numerical values depend on the modelled algorithms of the remaining errors, which may be different for GPS and Galileo. Additionally, SISA is characterizing the SIS error of nominal working Galileo satellites only. Nevertheless σ_i^2 and (SISA_i²+ $\sigma_{u,L,i}^2$) both are intended to characterize the ranging error within the range domain in a conservative way.

$$\underbrace{\left(SISA_{i}^{2}+\sigma_{u,L,i}^{2}\right)}_{Galileo} \triangleq \underbrace{\sigma_{i}^{2}=\sigma_{i,flt}^{2}+\sigma_{i,UIRE}^{2}+\sigma_{i,air}^{2}+\sigma_{i,tropo}^{2}}_{GPS+SBAS}$$
(11)

The fault-free allocation tree within the Galileo integrity concept implicitly equals the SBAS + GPS integrity concept except for the allocated confidence intervals and the representation of the final result. In (4), d_U is obtained in a manner similar to $\sigma_{u,V,FF}$ given by (10), shown by a detailed examination of the properties of the matrix M_{topo} in comparison to the general SBAS+GPS projection matrix. Similar analysis can be carried out for d_{major} used within (3) which equals to ξ_{FF} used within (8). Some deviations result because vertical precision is not monitored for *en route* procedures and along track errors are not monitored for precision approach procedures for SBAS + GPS.

With the transition of the SBAS + GPS integrity algorithm definition contained within RTCA DO-229 to the newer version, the rationale for the definition of the K values in equations (1) and (2) changes as well. As a consequence, only HPL and VPL are now conservative estimates, while the conservatism in the range domain is no longer guaranteed.

For the WAAS, this implies that the error distributions of the corrected pseudoranges are not conservative under all conditions. Nevertheless, the rationales of the SBAS + GPS integrity algorithm still hold true and, thus, the estimation of the combined integrity algorithm remains valid as well.

A final assessment of user integrity yields one major difference between the two concepts. While SBAS + GPS concept uses HPL and VPL (in meters) derived from fixed error allocations, Galileo uses the probability P_{HMI} with confidence intervals chosen by the user in terms of HAL and VAL (in meters).

In general, the protection levels and integrity risks at the alert limit are mathematically an inversion of the same context but cannot be compared directly here due to the different allocations. The discussion in the main article tries to overcome this problem by suggesting different solution strategies.

Working Papers continued from page 53

specified integrity risk can be granted and the VPL specifies the corresponding vertical value.

In general, the vector of the horizontal and vertical protection level is defined as

$$HPL = \max_{HAL \in R_{+}} P_{HMI,H} (HAL) \le IR_{H}$$

$$VPL = \max_{VAL \in R_{+}} arg P_{HMI,V} (VAL) \le IR_{V}$$

$$IR_{H} + IR_{V} \le IR$$
(12)

In the formulation of the GPS + SBAS integrity algorithm, the allowable integrity risk is split between the horizontal and vertical risk components in a fixed proportion, such that IR= $IR_H + IR_V$, and the probability function is separated into a horizontal and a vertical component.

$$\tilde{P}_{HMI}(HAL, VAL) = P_{HMI,H}(HAL) + P_{HMI,V}(VAL)$$

$$= P_{HMI}(HAL, \infty) + P_{HMI}(\infty, VAL)$$
(13)

This split is conservative, because the joint event

$$P_{HMI}((HP > HAL) \cap (VP > VAL)) \ge 0$$
(14)

disregarded in equation (13) is always positive. Together with the conservative use of the semi-major axis in (3), the inversion of the integrity risk computation is equivalent to the multiplication of the derived standard deviations d_U and d_{major} with the quantiles defining K-factors in (1) and (2).

Accordingly, the user integrity equations defined in Appendix J of RTCA DO-229, inverts the integrity risk under the following assumptions and simplifications:

- separation of the probability function in a horizontal and a vertical component
- disregard of the joint probability stated in (14)
- definition of a fixed split of the integrity risk in horizontal and vertical components
- consideration solely of the user geometry-dependent part of the total risk allocation.

In contrast to the user integrity algorithm defined in Appendix J, however, the Galileo baseline for user integrity algorithm specifies the integrity risk at the alarm limit directly — without inverting the respective CDFs with respect to the alarm limits for a specified integrity risk.

Nevertheless, some of the conservative simplifications deriving the K-factor formulation of the inverse integrity formulation within the SBAS user equations are implicitly performed in the Galileo user equations, too.

The separation of the probability function in a horizontal and a vertical component is given in equation (7). The equality in equation (7) has defining character, as

$$P_{HMI}(VAL, HAL) = P_{HMI,V}(VAL) + P_{HMI,H}(HAL)$$

$$- P_{HMI,V \cap H}(VAL, HAL)$$
(15)

The definition provided in (7) again is conservative, while (14) holds.

Together with these two fully identical assumptions, two

differences between the user integrity models remain.

- no fixed split of the integrity risk in horizontal and vertical components in the Galileo system
- additional integrity risk contributions of the faulty mode to the Galileo user integrity risk.

The fact that no fixed proportion exists to allocate integrity risk between horizontal and vertical components does not hinder the analytical solution of the inverse problem through implementation of the respective horizontal and vertical integrity risk-dependent K-factors.

However, the faulty mode component does not yield an analytical solution to the inversion problem because the pseudorange measurements are no longer assumed to be unbiased random variables. This results in a random error with a biased chi-square distribution in the horizontal position domain. The following section discusses possible means to solve the inversion problem.

Strategies for Solving the Inversion Problem

Figure 1 depicts the direct and indirect problem for one dimensional formulations. Because there is no fixed split for the integrity risk in horizontal and vertical components, an arbitrary number of solutions exist.

In order to guarantee a unique solution, therefore, one must consider imposing additional restrictions. Four possible inversion approaches include

- 1. defining a fixed allocation between horizontal and vertical integrity risk components
- 2. solving the horizontal component first
- 3. solving the vertical component first
- 4. computing the protection level using a geometry-dependent allocation

Similar to the fixed allocation method used by SBAS, fixing the horizontal and vertical risk allocations guarantees a unique solution. This approach splits the inverse problem into two distinct problems that can be solved separately by numerical means. The main advantage of this inverted algorithm is its similarity to the user integrity algorithm described in the RTCA standard, although it produces slightly worse availability results and causes lower computational efficiency.

The second approach is to compute in a first step the horizontal integrity risk associated with a given alarm limit. However, if this results in a computed horizontal integrity risk that exceeds the overall integrity risk, the algorithm terminates with the information that the demanded integrity level cannot be achieved. Otherwise, the computed horizontal integrity risk is allocated and also defines the vertical integrity risk expressed as

$$IR_{V} = IR - IR_{H} = IR - P_{HMI,H}(HAL)$$
(16)

The vertical component of the integrity risk IR_v together with the given vertical alarm limit, may then be used to solve the inverse problem unambiguously, where

$$HPL = HAL$$

$$VPL = \max_{VAL \in R} P_{HMI,V} (VAL) \le IR_V.$$
(17)





The proposed scheme shows a considerably better computational efficiency compared to the first method and causes a slightly better availability of the resulting system.

The third approach derives a unique solution in a manner similar to the second approach, with similar advantages of better availability and lower computational burden. In a first step, the vertical integrity risk at the given alarm limit is computed and compared to the overall integrity risk. The computed value then causes the horizontal integrity risk to be

$$IR_{H} = IR - IR_{V} = IR - P_{HMI,V}(VAL)$$
(18)

Together with the given alarm limit, the resulting protection levels are

$$VPL = VAL$$

$$HPL = \max_{\substack{HAI \in \mathbb{R} \\ HAI \in \mathbb{R}}} P_{HMI,H} (HAL) \le IR_{H}$$
(19)

The last inverting strategy designates the split of the integrity risk allocated between the horizontal and vertical part of the user equation to be proportional to the associated horizontal and vertical integrity risks at the alert limits. The integrity risk allocation follows

$$IR_{H} = \frac{P_{HMI,H}(HAL)}{P_{HMI,H}(HAL) + P_{HMI,V}(VAL)} \cdot IR$$

$$IR_{V} = \frac{P_{HMI,V}(VAL)}{P_{HMI,H}(HAL) + P_{HMI,V}(VAL)} \cdot IR$$
(20)

Equation (12) computes the respective protection levels. This last approach has a similar computational burden as the fixed allocation method, but the variable allocations result in better availabilities. A later section describes simulations of these approaches and discusses the different inverting strategies in more detail.

Because no analytical inverse of (8) exists, all methods must use a suitable root finding algorithm. Three common, wellknown methods — the Newton, secant, and bisection methods — have been considered for this purpose.

As both component functions $P_{HMI,H}$ and $P_{HMI,V}$ are composed of cumulative distribution functions and thus are continuous, differentiable and strictly monotonous, the maximum of (17) and (19) is uniquely realized with equality

$$P_{HMI,X}(XPL) = IR_X$$
⁽²¹⁾

To apply standard root finding algorithms, we must shift the integrity risk functions by the allocated integrity risk.

$$P_{s}(XPL) = P_{HMI,X}(XPL) - IR_{X}$$
⁽²²⁾

The Newton method uses the integrity risk function's derivative beside the shifted version of the function itself. Because the integrity risk function given in (8) contains cumulative distribution functions, its derivative is a linear combination of the corresponding probability density functions. Unfortunately the Newton method does not always converge. In particular flat slopes can cause divergence or very slow convergence. For sufficiently good initial estimations, the iteration converges quadratically.

The secant method is also applied to the shifted integrity risk function (22). Using an iterative process, one starts with two arbitrary initial guesses for x_o and x_i , not necessarily embracing the root of P_s . In each iteration step, the secant is used to extrapolate the function and its intersection with the abscissa is computed and used as next guess.

The secant method is a derivative of the Newton method and, also, does not necessarily converge. The method converges locally in a super linear fashion, although slower than the quadratic convergence of the Newton method.

The bisection method also uses the shifted version of the integrity risk function and starts with an initial guess of an interval $[x^l, x^u]_1$ embracing the root of the function, thus $P_s(x_1^l) \cdot P_s(x_1^u) < 0$. In each iteration, the interval spanned by the preceding boundaries is bisected and the part containing the zero crossing of the shifted integrity risk function is chosen for the next step. This method converges linearly, and the solution's accuracy depends only on the length of the last interval.

Using the bisection method is recommended to invert both the horizontal and vertical integrity risk functions, as it is difficult to find appropriate initial guesses following in fast converging iterations for the Newton and secant method.

Deriving Integrity Risks for GPS + SBAS Measurements

As already mentioned the formulation of the integrity risk at the alarm limit is referred to as the direct problem. For reasons of comparability, this section will summarize how to express the protection level formulation of GPS + SBAS into integrity risks at the alarm limit. Different assumptions for, respectively, en route nonprecision approach and precision approach lead to two different formulations.

En Route Integrity Risk Formulation. For each line of sight (LOS), equation (6) conservatively estimates the variance of the SBAS + GPS measurement error using a normal distribution. Using a weighted least squares method, where equation (23) describes the weight matrix



the law of error propagation determines the variance of the vertical error to be d_u as defined in (4). Equation (24) calculates the vertical component of the geometry-dependent integrity risk for a given alarm limit VAL. For *en route* operations the vertical position component is not monitored corresponding to "VAL = ∞ ".

$$P_{HMI,V}(VAL) = 2 \cdot \int_{VAL}^{\infty} \frac{1}{d_u \sqrt{2\pi}} \cdot \exp\left(-\frac{1}{2}\left(\frac{x}{d_u}\right)^2\right) dx$$
(24)

For the horizontal error estimation, one computes the semi-major axis of the error ellipse following (3) and (4). In the worst-case estimates, both coordinate directions are normally distributed with a standard deviation equal to d_{major} . Thus, the horizontal error follows a Rayleigh distribution with Rayleigh density d_{major} . The resulting integrity risk can equivalently be expressed using a chi-squared cumulated density function.

Equation (25) describes the horizontal component of the geometry dependent integrity risk, for a given horizontal alarm limit, HAL. The horizontal error is modelled as the norm of a vector of two normal distributed error variables X, Y with standard deviation d_{maior} .

$$P_{HMI,H}(HAL) = 1 - \chi_2^2 cdf\left(\frac{HAL^2}{d_{major}^2}\right)$$
(25)

Precision Approach Integrity Risk Formulation. Using equation (6) one computes the LOS error variance. Applying the weighting matrix (23) and error ellipse matrix (4), the variance of the height error computes again to d_{u} , and equation (24) yields the vertical geometry dependent integrity risk.

The horizontal error ellipse's semi-major axis is estimated using equations (3) and (4), but the user's position is only bound in one dimension, as the tolerable along-track errors are several magnitudes greater compared to the associated tolerable cross track errors. The lateral error is assumed to be normal-distributed with a standard deviation equal to d_{major} . Equation (26) describes the horizontal integrity risk contribution.

$$P_{HMI,H}(HAL) = 2 \cdot \int_{HAL}^{\infty} \frac{1}{d_{major}\sqrt{2\pi}} \cdot \exp\left(-\frac{1}{2}\left(\frac{x}{d_{major}}\right)^2\right) dx \quad (26)$$

The geometry-independent integrity risk contribution may be obtained by the difference of the allowable integrity risk and the geometry dependent integrity risks associated with the given inflation factors in (1) and (2).

Combined Use of Integrity Information

When considering the combined use of integrity information provided by SBAS + GPS and Galileo, one could think of using only SISA within the SBAS + GPS integrity concept by exploiting the similarities leading to (11). This approach would neglect SISMA, because there is no similar parameter within the SBAS + GPS integrity concept on the user side. In any case, this approach is faulty by design due to fundamental system definitions. As previously mentioned, the integrity concept of SBAS + GPS defines all satellites as healthy on the user side, when complete integrity information exists. Satellites must be designated unhealthy in order to be excluded. Additionally, the SBAS ground segment holds full responsibility for detecting any hazardous event, except for the user's use of, for example, jump detectors, and RAIM.

On the other hand, the Galileo baseline integrity concept treats, for each instant of time, one of the satellites designated healthy by the ground monitoring segment as faulty. SISMA describes the distribution of the SIS error within the faulty mode allocation tree, in combination with SISA. SISA and SISMA combine together with an *a priori* probability of missed detection to estimate the worst-case bias for faulty measurement.

Unfortunately, the discrepancy between the Galileo and SBAS + GPS system definitions cannot be resolved on the user side within a reasonable order of magnitude of the assumed SIS errors. As a result, the approach of using only SISA within the SBAS + GPS integrity concept is not practical.

The parallel calculation of integrity using the SBAS + GPS and Galileo integrity algorithms independently, followed by an *a posteriori* integration of results appears to be unsuitable. This is mostly due to the fact that the integration of the two independent results means that either the two results will be averaged in some way, or a combined solution based on internal algorithm data, such as userto-satellite geometry, will be used for the combination.

Averaging the two results provides an arbitrary and worse outcome compared to a true integrated integrity algorithm. Similarly, the combined a posteriori solution has no benefits compared to an integrated algorithm and also results in a worse solution.

Nevertheless a user not interested in a combined integrity solution may use one of the independent solutions as the main integrity source, treating the second as an additional consistency test. Finally, one should note that the general statement, "The more integrity data, the better the knowledge of the integrity of the navigation solution," does not necessarily mean that the more satellites used within the integrity algorithm, the better the derived measure of integrity.

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the protection level gets smaller) by its geometrical contribution, the measure of integrity simultaneously degrades by the probability that this satellite is faulty. This is particularly true in the case where the benefit of the geometrical contribution is small and may even lead to a situation where adding a satellite to the calculation actually degrades the measure of integrity.

Combined Algorithm

A combined integrity algorithm can be formed by using the data provided by the SBAS satellites within the Galileo integrity equations. SISMA provides the only data not available to the user from the SBAS satellites compared to the integrity information from the Galileo satellites. Within the current baseline Galileo integrity concept, SISMA is needed for the calculation of the "faulty" mode allocation tree.

On the other hand, according to the RTCA MOPS, the SBAS + GPS integrity concept states that all GPS satellites that are spuriously considered healthy by the SBAS ground segment only cause a fixed, geometry-independent integrity risk.

In terms of the Galileo integrity concept, this equals the definition of $P_{fail,sat}=0$ for all GPS satellites considered healthy by the SBAS ground segment. This calculation assumes at the same time that, for the entire set of GPS satellites, a geometry-independent integrity risk comes from the different fault allocation trees used by Galileo

and GPS + SBAS and is added to the outcome of the combined algorithm. By defining a $P_{fail,sat} = 0$ for all GPS satellites with valid integrity information, the whole "faulty" mode allocation tree will become zero.

Therefore, SISMA is not needed for GPS satellites used within the current baseline Galileo integrity concept, because the computed integrity risk

includes the geometry-independent portion.

The combined integrity algorithm is used to assess the measure of integrity in the combined navigation solution using SBAS + GPS and Galileo ranging data. The use of SBAS + GPS and Galileo integrity data within a combined integrity algorithm requires knowledge of the relationship between both systems' coordinate reference frames and time systems. The transformation parameters between the two systems can be determined in advance with sufficient accuracy.

The Galileo navigation messages will contain the time difference between GPS and Galileo timescales. Nevertheless, for integrity algorithms this timescale difference may be determined more suitably from the satellite measurements themselves. Because this difference is not currently known, a second clock error is introduced to the integrity algorithm's design matrix H in (9). The baseline Galileo integrity algorithm remains otherwise unchanged.

Naturally this approach reduces satellite redundancy by one, but the ability to use all available satellites from both GNSSs in the combined algorithm outweighs the cost of slightly less redundancy. The result of the combined integrity algorithm describes the risk at an alert limit, which can be converted to protection levels as described in the previous section.

WORKING PAPERS

Integrity Simulation Tool

To be able to estimate the performance of a combined GPS+SBAS and Galileo integrity algorithm, an Integrity Simulation Tool (IST) was developed and implemented in C++ with a platform-independent design in order to accommodate several working tasks. The required functionality of the IST includes:

- SBAS data processing conforming to RTCA DO-229D standards
- SBAS performance estimation on a global scope
- Galileo integrity concept performance estimation
- Combined algorithm performance estimation.

The IST provides additional functionality, including raw measurement generation, random measurement degradation, and data interfaces between simulated space/ground and user segments.

Plain Kepler elements or GPS ephemeris describes the satellite constellations. The user segment is described as a uniformly distributed grid, while the ground segment is described as a list. A configuration file contains SBAS and Galileo integrity information, modeled in simplified form or provided by the ground segment object.

Different atmospheric models, including TropGrid, Saastamoinen and Neill blind models for the troposphere are implemented. Ionospheric delay follows either the Klobuchar or NeQuick model. Random variations in satellite motion, tropospheric, and ionospheric delay, are captured by a variety of random walks and bounded random walks. The generation of raw measurements is only performed for the ground segment objects.

An Integrity Tool application, capable of carrying out calculations for all mentioned integrity algorithms, complements the IST. This application can operate in stand-alone mode, supplied by a single user position data of the IST or a different application via the user datagram protocol (UDP), or it can also run as an integrated component within the software receiver developed at the University FAF. **Figure 2** shows a screen capture of the Integrity Tool connected to the IST.

Simulation Results

Based on Monte Carlo simulations using the IST the combined integrity algorithm's performance was validated. All simulations used several basic settings, held con-



Simulation Tool

stant within the scope of each simulation. The simulation duration is 10 days for each timeline scenario. The simulated Galileo satellite constellation uses a 27/3/1 Walker constellation, while the GPS satellite constellation incorporates real ephemeris data gathered in November 2008.

This design produces an arbitrary inertial shift between constellations due to the unknown positions in the Galileo constellation. Nevertheless, due to the different drift of the orbital planes no fixed inertial shift will occur between both satellite constellations. The simulated satellite orbits do not try to account for perturbation effects.

Ionospheric noise is modelled proportional to the slant delay, derived from the Klobuchar delay model. The Klobuchar parameters are held fixed for the duration of the simulation.

All simulations are carried out for a defined mesh grid of observer positions. The simulations assumed that all observers between $\pm 80^{\circ}$ latitude ($10^{\circ} - 170^{\circ}$ colatitude) have access to at least one SBAS satellite. For Galileo satellites the HAL is set to 12 meters and the VAL is set to 20 meters, as taken from the paper by V. Oehler et alia (2006). The default SISMA value is set to 0.80 meters for all Galileo satellites.

Single-Epoch Analysis. The results of the first epoch of simulated 10-day scenarios are presented exemplarily, where all epoch analyses show similar results. The ionosphere noise is modelled based on the estimated slant delay multiplied with a dedicated ionosphere factor. The maximum zenith delay for this simulation was of about six meters over South America.

Figure 3 presents calculated HPL values. **Table 1** describes the performance assumptions of the Galileo and GPS systems, where equivalent simulation parameters for both systems are chosen in order to avoid an unfair comparison between the theoretical performance of the future Galileo satellites and the performance of the current GPS satellites.

Although the SBAS + GPS integrity algorithm performs well in this case, the combined integrity algorithm provides much better HPLs.

It has to be emphasized that the increased number of satellites within the combined algorithm prevents the user from suffering from weak constellation gaps, as illustrated by the much more uniform HPL distribution for the combined algorithm in the left upper part of Figure 3.

To provide an estimation of the influence of the orbit and clock precision SISA on the overall Galileo system performance, **Figure 4** shows the vertical integrity risk at an alarm limit of 20 meters using a logarithmic scale. SISA values are assumed to range between 0.85 meters to 3 meters, while the receiver noise is ignored and the Galileo ionosphere factor is set at 0.02.

In the case of degraded SISA values, the performance of the Galileo integrity







FIGURE 4 Comparison of Galileo only vertical integrity risks for different SISA values

baseline algorithm is significantly less compared to the nominal performance based on a SISA value of 0.85m. Following Equation 11, the integrity equation uses the overall standard deviation,

Receiver noise (GPS and Galileo)	1.00 m	
Tropospheric noise	0.05 m	
Orbit and Clock noise GPS $(\sigma_{ ext{UDRE}})$	2.00 m	
Orbit and Clock noise Galileo (SISA)	2.00 m	
GPS ionosphere factor	0.02	
Galileo ionosphere factor	0.02	
TABLE 1. Main properties, Scenario 1: single epoch		

composed of SISA and the remaining errors $\sigma_{u,L,i}$.

Thus, the same result holds when remaining errors vary, e.g. the receiver noise error. This result offers insight into the algorithm's parameter dependencies and the influence of the remaining errors and possible degraded SISA and SISMA values. The simulation also offers a method for selecting appropriate upper bounds for suitable models and estimating the maximum allowed magnitude of the remaining error components such as receiver noise and multipath.

Timeline Analysis. Timeline analyses

for a worldwide mesh grid are based on 10-day scenarios, using the repeat period for the future Galileo space segment. Using a 600-second time step for the presented calculations, results in a total number of 1,440 epochs. The ionosphere was modelled as in the single epoch analysis, where the maximum zenith delay wanders over the southern hemisphere at a latitude of 25° S (co-latitude of 115°).

Figure 5 compares the HPL values calculated by the combined integrity algorithm with the Galileo integrity algorithm; **Table 2** describes the main properties of this second scenario. The results are derived by using the fixed inversion scheme as described earlier. The upper row of Figure 5 shows the mean HPL values for every grid point and provides a rough estimation of the general order of magnitude for the HPL values. The mean HPL does not yield any statement regarding integrity violating events.

One can clearly see that the averaged performance of the combined algorithm is better than that of the Galileo algorithm alone. The same or even worse performance would result from the HPLs derived using the SBAS + GPS integrity algorithm.

The system performance requirements for the Galileo safety-of-life (SoL) service define a HAL of 12 meters to be fulfilled for an integrity and continuity risk as described by V. Oehler et alia (2005). The second row of Figure 5 provides the percentage of epochs per grid point in which the HPL exceeds 12 meters.

As expected, no violating events occurred within the simulated epochs for either the Galileo or the combined algorithms. From this, we conclude that under nominal conditions, the Galileo SoL Service will be able to fulfil the required system performance.

The third row of Figure 5 presents the results in case the HAL threshold is "artificially" lowered to nine meters, where different scales are used for the Galileo only HPLs and the combined HPLs. Even if a reduction of the HAL by 25 percent seems to be moderate, availability remains far out of the reach

WORKING PAPERS





FIGURE 6 Munich plots for different inversion strategies (relative values additionally scaled logarithmic to the base of 10; plot granularity 1 meter)

Tropospheric noise	0.05 m	
Orbit and Clock noise GPS ($\sigma_{ ext{UDRE}}$)	2.00 m	
Orbit and Clock noise Galileo (SISA)	0.85 m	
Galileo SISMA	0.8 m	
GPS ionosphere factor	0.2	
Galileo ionosphere factor	0.02	
TABLE 2. Main properties, Scenario 2: timeline analysis		

of SoL service design specifications.

Thus, the Galileo only integrity algorithm would not be applicable under this artificially lower requirement. Here again, the SBAS + GPS integrity algorithm would deliver even worse results than the Galileo only algorithm, as the lower performance is out of this algorithm's specifications, too.

Nevertheless, the combined integrity algorithm once again would provide significantly better performance, as depicted at the left hand side of the third row. From this, we conclude that the combined integrity algorithm provides user integrity under much more demanding requirements. With regards to possible needs for better HPL's and VPL's, e.g. for higher order precise landing operations, a combined algorithm has a bigger potential than a single system integrity algorithm.

The HPL and VPL values derived from integrity risks using different inversion strategies are depicted within Figure 6 using so-called Munich Plots. The plots show color coded the frequency of all possible HPL / VPL combinations using the Galileo-only integrity algorithm. To demonstrate the effects of the different inversion schemes, integrity-violating events have been simulated by intentionally increasing the assumed Galileo receiver noise within the specific simulations. The depicted values are normalized so that the sum over all values per diagram is 1. If either a VPL of 20 meters or an HPL of 12 meters is exceeded, an integrity-violating event is assumed.

The total number of violating events and indirectly the relative change between the different inversion strategies naturally depends on the bounds and remaining margins. Therefore, the following results shall be understood as generalities where the absolute numbers are not important. The upper left plot of Figure 6 show the result of a fixed split between the integrity risk's horizontal and vertical components, analogous to the fixed allocation defined for SBAS.

This inversion strategy results in around 210,000 events in which the bounds have been violated. The variable allocation scheme depicted in the upper right plot shows a better performance and the number of violating events was reduced by nearly 5,000.

Even if this seems to be a small reduction in the number of total violation events, one has to keep in mind that it was accomplished by removing only the fixed allocation and is based on the same set of measurements.

Within the "real world" this would translate into better integrity performance for a user employing the integrity split best suited for a particular application and geometry.

The extreme cases are depicted within the lower row of Figure 6 for users of either the HPL or the VPL, respectively, who have an absolute preference against the other bound. (The inversion strategies for these scenarios were presented earlier in the solution strategies section.)

In these cases, a reduction in violating events compared to the variable allocation strategy occurs. For the VPL first strategy, the bar on the right side of the diagram is based on the fact that for some grid points the allocated maximum integrity risk was less than the calculated vertical integrity risk associated with the given alarm limit. In these cases the HPL was set to infinity, and the overall allocation was used for the VPL.

Conclusions

The simulations and analyses presented in this column show that the planned performance parameters of the Galileo system now under development are challenging and highly dependent on the clock and orbit accuracy.

Inverting strategies for shifting protection level formulations to integrity risk formulations provide a better comparability of Galileo integrity with SBAS + GPS integrity. Using the most simple inversion strategy — inversion with fixed allocations — one has to pay the price of gradually reduced system availability. Inverting strategies with variable allocations show better results.

The conservative joint of the different integrity risk allocation trees results in an additional additive and geometryindependent integrity risk component for all GPS satellites. The simulation results demonstrate that this additive term in the combined algorithm does not deplete the geometry and redundancy induced advantages. Consequently, combined use of integrity information outperforms either single system used alone.

Beginning October 2009, the EGNOS open service was declared to be operational. Although it provides the same information compared to the future EGNOS SoL service, the EGNOS open service definition document underscores the fact that SoL users should not use EGNOS for safety critical purposes, until the EGNOS SIS and its operator are certified for SoL purposes. The EGNOS certification process will be closed out mid-year, not allowing until this time the possibility to define SBAS procedures in Europe. This demonstrates the discrepancy between provided and certified integrity data. The combined use of different integrity sources presented in this column shows that enhancements in both availability and achievable protection levels can be expected.

Nevertheless it is the solely decision of all involved service providers to jointly define and certify combined integrity processing schemes, combined equipment regulatory and combined procedures. Also, the providers have to jointly keep the liability for the combined system.

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