GNSS Solutions:

Orbital precession, optimal dual-frequency techniques, and Galileo receivers

"GNSS Solutions" is a regular column featuring questions and answers about technical aspects of GNSS. Readers are invited to send their questions to the columnists, **Professor Gérard Lachapelle** and **Dr. Mark Petovello**, Department of Geomatics Engineering, University of Calgary, who will find experts to answer them. Their e-mail addresses can be found with their biographies at the conclusion of the column.

Is it true that the GPS satellite geometry repeats every day shifted by 4 minutes?

It is true that the GPS satellite orbits were selected to have a period of approximately one half a sidereal day to give them repeatable visibility. (One sidereal day is 23 hours, 56 minutes, and 4 seconds long or 236 seconds shorter than a solar day.) However, because of forces that perturb the orbits, the repeat period actually turns out to be 244 to 245 seconds (not 236 seconds) shorter than 24 hours, on average, and changes for each satellite.

The selection of a half sidereal day orbit causes the satellite ground track and the satellite visibility from any point on earth to be essentially the same from day to day, with the satellites appearing in their positions approximately 4 minutes (236 seconds) earlier each day due to the difference between sidereal and solar days. This was a particularly useful property in the early days of GPS when session planning was important to ensure adequate satellite coverage. With this easily predictable coverage, GPS users could schedule repeatable campaign sessions well in advance just by shifting their experiments forward each day by 4 minutes.

Operationally, the GPS satellite orbits are actually specified in terms of the repeatability of the longitude of the ascending node (the longitude at which the orbit crosses the equator moving northward), a term that is kept within ±2 degrees of a nominal value. This essentially constrains the ground track of the satellite to repeat to within about ±222 kilometers in an east-west direction at the equator. When a satellite orbit goes outside this range, the GPS operational control segment makes an orbit adjustment.

For an ideal Keplerian orbit, the repeating ground track can be established by setting the semimajor axis

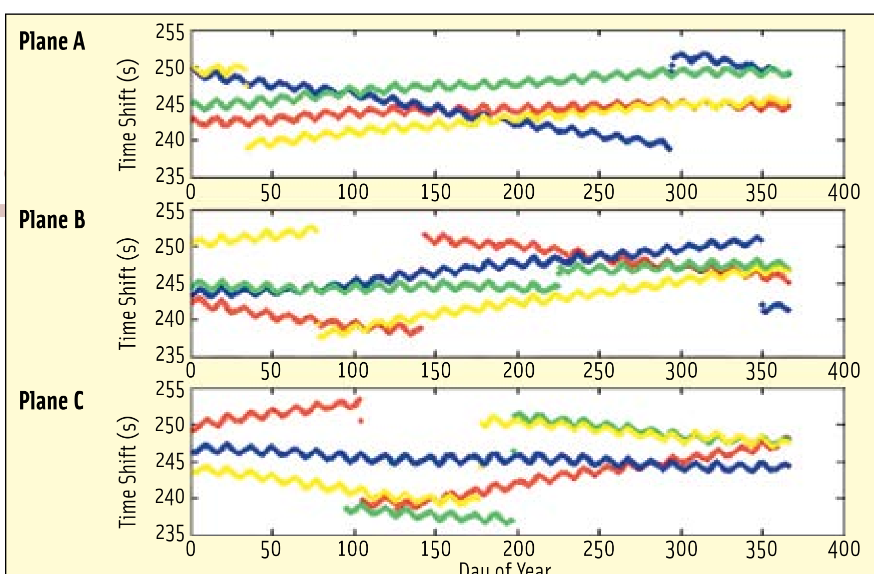


FIGURE 1 Daily time shift of GPS satellite ground track repeat relative to 24 hours based on broadcast ephemeris data for 2004.

of the orbit (equal to the radius for a perfectly circular orbit) such that the period of motion equals one half the sidereal day:

$$P = 2\pi\sqrt{a^3/\mu}$$

$$a_{repeat} = \left[\mu\left(\frac{\frac{1}{2}P_{sidereal}}{2\pi}\right)^2\right]^{1/3} = 26{,}561.74\ km$$

where P is the orbital period, a is length of the semimajor axis, and μ is the product of the universal gravitational constant and the mass of the Earth and is equal to 398600.5 km^3/s^2. In this ideal case the satellites would appear in the same place in the sky 236 seconds earlier each day.

Less Than Perfect Orbits

In reality, each satellite orbit is "perturbed" or modified by the non-central gravity field of the earth; the gravitational attraction of the sun, moon, and other planets; solar radiation pressure; and atmospheric drag. Earth oblateness (the bulge around the equator) has the largest effect on the ground track repeat at the GPS orbit altitude, producing a westward drift of the longitude of the ascending node of 14.665 degrees per year.

To compensate for this, GPS system operators set the average semimajor axis of the satellite orbits slightly low such that the orbital period is about 4 seconds faster than a sidereal half-day. With two orbits per day, the time shift of the daily repeat for most satellites in the constellation is thus about 244 seconds (i.e., eight seconds larger than 236 s).

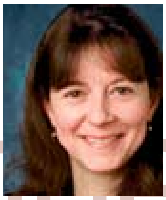
Figure 1 shows the time shift of the ground track repeat computed on a daily basis for the nominal A, B, and C Plane GPS satellites during 2004. These satellites are seen to have a typical shift value of 244 seconds, plus a fairly linear drift, some small oscillations, and an occasional step. The linear or "secular" drift is due to a near resonance of the GPS orbits with the tesseral harmonics of the Earth's gravitational field. The small amplitude oscillations occur twice monthly due to perturbations of the orbits caused by the moon. The abrupt steps in the time shift are due to satellite orbit maintenance maneuvers that can be identified in the GPS NANUs (Notice Advisory to Navstar Users). Satellites that are being substantially repositioned do not have precisely repeating orbits.

A more precise way to find the best time shift for each satellite is to compare the line-of-sight vectors at a location of interest based on the broadcast or precise ephemeris data. This approach is a good idea if you are trying to use satellite repeatability to filter out multipath. However, it means that you may encounter a slightly different time shift for each epoch. A straightforward way to find the shift is to compute the line-of-sight unit vectors on each day, for example:

$$\hat{\mathbf{e}} = \cos(el)\sin(az)\,\hat{\mathbf{E}} + \cos(el)\cos(az)\,\hat{\mathbf{N}} + \sin(el)\,\hat{\mathbf{U}}$$

Then, for each epoch on the current day, find the unit vector on the previous day that maximizes the dot product (which is equal to the cosine of the angle between them).

PENINA AXELRAD AND KRISTINE M. LARSON



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Kristine M. Larson is professor of aerospace engineering sciences at the University of Colorado at Boulder. Her research focuses on the development of high precision methods in GPS and their application to geophysics.

How can dual frequency code and carrier measurements be optimally combined to enhance position solution accuracy?

The smoothing of GPS code pseudorange measurements with carrier phase measurements to attenuate code noise and multipath is a well-established GPS signal processing technique. Unlike carrier phase real time kinematic (RTK) techniques, carrier-smoothed code (CSC) positioning solutions do not attempt to resolve carrier phase ambiguities. As a result, they offer a number of design and operational advantages for those applications that do not require RTK accuracies.

Ionospheric effects are a limiting factor in how much smoothing of pseudorange errors can be accomplished with single-frequency measurements. The use of dual-frequency code and carrier measurement combinations in CSC processing to attenuate pseudorange errors and as a precursor for carrier phase ambiguity resolution has gained increasing importance, particularly with the availability of all-in-view dual-frequency GPS receivers in the survey and military markets. Interest in these techniques will increase with the advent of additional GNSS signals as the result of GPS modernization and implementation of Galileo, along with the proliferation of differential services.

Figure 1 illustrates basic CSC signal processing. CSC processing can be implemented in other ways, but all yield essentially the same results. The dual frequency (L1 and L2 respectively) pseudorange measurements, ρ_1 and ρ_2, are linearly combined to obtain a

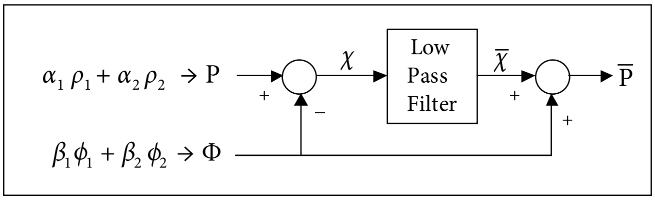

FIGURE 1 **Block diagram of Carrier Smoothed Code (CSC) Signal Processing.**

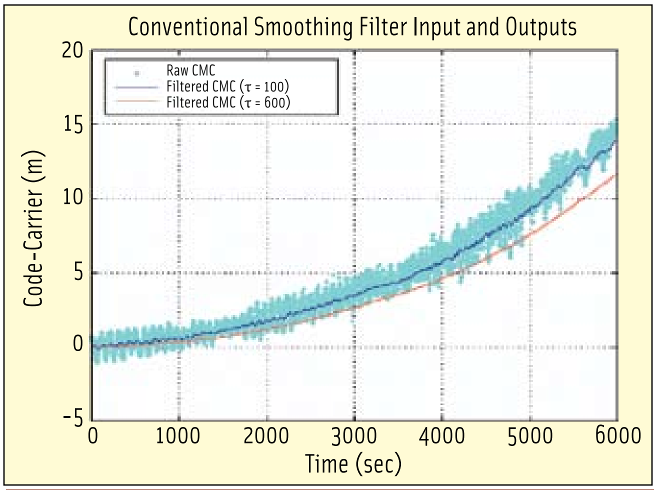

FIGURE 2 **Raw and filtered CMC signals for a single frequency data set.**

generalized pseudorange measurement, P, using scaling factors α_1 and α_2. Similarly, a generalized carrier phase measurement, Φ, is formed from the corresponding carrier measurements, ϕ_1 and ϕ_2. All carrier phase measurements are in units of length, e.g., meters. To maintain the line of sight (LOS) range information, we require that the coefficients add to one, i.e.,

$$\alpha_1 + \alpha_2 = 1 \qquad \beta_1 + \beta_2 = 1 \qquad (1)$$

The function of the low-pass filter in Figure 1 operating on the code-minus-carrier (CMC) signal, X, is to attenuate the code noise and multipath while providing an estimate of the offset of the carrier phase measurement from the code pseudorange. The smoothed generalized pseudorange, $\bar{\mathrm{P}}$, is formed by correcting the generalized carrier phase measurement with the estimated carrier phase offset in the filtered CMC.

Single Frequency CSC

The simplest form of CSC processing is with single frequency measurements, e.g., $\alpha_1 = \beta_1 = 1$, $\alpha_2 = \beta_2 = 0$ for L1 processing. For example, **Figure 2** shows the raw and filtered CMC signals in a typical L1 data set. Note that the CMC has been initialized to zero for plotting purposes. In the single-frequency case, the CMC signal contains twice the code ionospheric error because the ionosphere *delays* the code modulation but *advances* the carrier phase. When the ionosphere varies with time, the low-pass filter inevitably induces some degree of lag. For a first-order filter with time constant τ, and a linear ionosphere time variation of dI / dt, the filtered CMC signal is biased from the unsmoothed CMC by $2(dI / dt)\tau$. Figure 2 shows this clearly in the $\tau = 600$ seconds case represented by the red curve. This ionospheric divergence bias also corrupts the smoothed pseudorange measurement.

The effect of ionospheric divergence places a number of limitations on the utility of single-frequency CSC processing. The pseudorange biases induced by long smoothing time constants implies that no benefit results from smoothing beyond 20 seconds or so in standalone applications because the decrease in noise and multipath is offset by an accompanying ionospheric divergence bias.

In code differential systems the smoothing bias can be cancelled if the reference station and rover receiver employ identical filters. Although we can readily achieve this in steady state operation, increased errors occur during smoothing filter convergence after signal acquisition or reacquisition, or after resetting the filter following carrier phase cycle slips. The filter convergence takes two to three time constants; so, smoothing constants are typically limited to around 100–200 seconds to minimize the effect of filter resets on operational availability. Ground-based augmentation systems for aircraft precision approach and landing have settled on $\tau = 100$ seconds as a compromise value.

Divergence-Free CSC Processing

The proper use of dual-frequency code and carrier measurements can free the system designer from worrying about the effects of ionospheric divergence. The key idea is to choose α_1, α_2, β_1, and β_2 as shown in Figure 1 to cancel the ionosphere error on the CMC signal, which eliminates the effects of ionospheric divergence in the CSC processing. The unsmoothed pseudorange ionospheric error will remain in the smoothed output, but no ionospheric divergence bias will be present. To satisfy the divergence-free (DF) condition we must have

$$(\alpha_1 + \beta_1) f_2^2 + (\alpha_2 + \beta_2) f_1^2 = 0 \qquad (2)$$

where f_1 and f_2 are the carrier frequencies. An infinite number of possible DF CSC configurations satisfy the constraints in equations (1) and (2). We now present some specific special cases of DF CSC based on particular choices for α_1, α_2, β_1, and β_2.

L1 and L2 DFS. Choosing $\alpha_1 = 1$, so that $\mathrm{P} = \rho_1$, yields the L1 DF CSC; similarly, choosing $\alpha_1 = 0$, so that $\mathrm{P} = \rho_2$, yields the L2 DF CSC. The CSC filter inputs in these cases are:

$$\mathrm{P}_1 = \rho_1, \quad \Phi_1 = \frac{f_1^2 + f_2^2}{f_1^2 - f_2^2}\phi_1 - \frac{2 f_2^2}{f_1^2 - f_2^2}\phi_2$$

$$\mathrm{P}_2 = \rho_2, \quad \Phi_2 = \frac{2 f_1^2}{f_1^2 - f_2^2}\phi_1 - \frac{f_1^2 + f_2^2}{f_1^2 - f_2^2}\phi_2$$

Figure 3 shows the raw and filtered CMC for L1 DF measurements corresponding to the data set shown in Figure 2. Note that the DF CMC does not drift off as in the single-frequency

case. The extended smoothing constant can be seen to significantly reduce the pseudorange errors over the 100-second case without incurring any divergence bias error. This plot illustrates one of the major benefits of DF CSC processing in differential applications: the reference station and rover processing can be completely decoupled and optimized for their specific environments. For example, a stationary reference station can use extended smoothing time constants to attenuate multipath, whereas a dynamic rover may wish to use a shorter time constant for faster convergence.

Ionosphere-Free Smoothing. A special case of DF CSC is obtained by eliminating the ionosphere in both P and Φ individually, rather than only in the CMC as discussed previously. This is achieved with the following parameters:

$$\alpha_1 = \beta_1 = \frac{f_1^2}{f_1^2 - f_2^2} \triangleq \alpha_{IF}$$

$$\alpha_2 = \beta_2 = \frac{-f_2^2}{f_1^2 - f_2^2}$$

Wide-Lane Carrier Phase. A standard technique in RTK systems is to form *wide-lane* carrier phase combinations to improve the possibility of integer ambiguity fixing. We can demonstrate a DF pseudorange combination that corresponds to the wide-lane carrier; the generalized pseudorange and carrier phase measurements in this case are:

$$P_{NL} = \frac{f_1}{f_1 + f_2}\rho_1 + \frac{f_2}{f_1 + f_2}\rho_2 \triangleq \alpha_{WL}\,\rho_1 + (1-\alpha_{WL})\rho_2$$

$$\Phi_{WL} = \frac{f_1}{f_1 - f_2}\phi_1 - \frac{f_2}{f_1 - f_2}\phi_2 \triangleq \beta_{WL}\phi_1 + (1-\beta_{WL})\phi_2$$

The code combination that is required to achieve DF with the wide-lane carrier phase actually corresponds to what is often termed the *narrow-lane* pseudorange code combination, and we have denoted this accordingly.

In RTK applications, double differencing DF narrow-lane CSC pseudorange measurements provide a direct means to initialize the wide-lane ambiguity search — which is the basis for geometry-free ambiguity methods. Even with ambiguity space search methods, the long smoothing possible with DF CSC processing can be applied separately to reference station and rover measurements, suppressing code multipath as much as possible prior to ambiguity search processing, since code errors limit the speed and reliability of on-the-fly ambiguity fixing. (This approach seeks to smooth at the reference station so that, when a rover comes into range, the code multipath has already been suppressed as much as possible.) We can also show that the narrow-lane pseudorange combination yields a reduction in multipath and noise errors compared to the single-frequency measurements prior to smoothing.

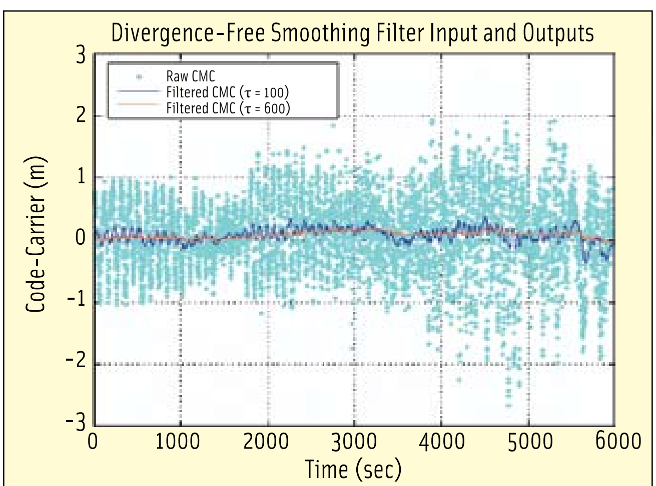

FIGURE 3 L1 Divergence-Free raw and filtered CMC.

Summary

Divergence-free carrier-smoothed code processing offers a number of performance and operational advantages over conventional single-frequency CSC. The various DF modes discussed here have numerous applications in both standalone and differential navigation systems. The ability to smooth pseudorange measurements for extended periods without concern for ionospheric divergence yields improved accuracy over conventional processing. Modernized GNSS developments will permit wider use of DF CSC processing as the result of increased access to multiple GNSS signal frequencies.

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What is the availability of Galileo receivers?

With the launch of the GIOVE-A (Galileo In-Orbit Validation Element - A) Galileo test satellite in December last year, the European Galileo satellite navigation system is making progress. How will we be able to recognize the benefits of Galileo? We will require enough Galileo satellites to make a difference when used with GPS alone, and we will require dual-mode Galileo/GPS receivers.

First, let us recap what Galileo will provide to users. And second, let us summarize what benefits we can expect to see, not only from Galileo alone but from a combined GPS/Galileo constellation of approximately 60 satellites.

Galileo will offer several worldwide service levels, including open access and restricted access for various segments of users. These services include:

- a basic Open Service (OS), supplied free of charge to the general public.

- a for-fee Commercial Service (CS) for professional, high-precision applications
- a Safety of Life Service (SoL) providing enhanced accuracy and integrity for safety-critical applications such as aircraft approach and landing a Search and Rescue (SAR) service
- an encrypted Public Regulated Service (PRS), for military and para-military users.

These services are mostly compatible with existing GPS services, and users are expected to demand the added reliability, integrity, and functionality that a combined GPS and Galileo receiver will provide when signals from both systems are used together. Anticipated advantages to a user of an integrated Galileo/GPS receiver include:

- Twice as many satellites provide twice the probability of receiving good signals with favorable geometry when visibility is reduced or blocked.
- Vehicle operators in urban environments will see more signals, more often and suffer less from signal blockage.
- Surveyors will have higher accuracy measurements, more consistently.
- Automated guidance for agricultural sprayers, combines and harvesters will be more accurate, and signal reception will be improved, reducing signal outages.
- Difficult inshore navigation on rivers and canals will be safer and more reliable.
- Aircraft en route navigation, final approach, and landing will have far greater signal redundancy, which could well result in improved safety margins and reduced decision heights for landing.

Even with the addition of only a few Galileo satellites — GIOVE-A and B and the four planned IOV (In-Orbit Validation) satellites — coverage will be improved for all users, and we will start to see some of these benefits. Gradually, as the constellation grows and dual-mode receivers see more and more GPS and Galileo satellites, these benefits will increase and become more pervasive. Eventually, as many as 60 GPS and Galileo satellites may be seen continuously all over the world.

Galileo receiver development has been under way for some time, well in advance of an available satellite constellation. The drivers for this activity include:

- European Space Agency sponsorship of receivers needed for use in GIOVE and IOV satellite testing, and for subsequent signal validation. This includes the Galileo Experimental Test Receiver (GETR) receiver built by Septentrio, and two teams of companies managed by Septentrio and Thales Avionics, which are developing independent versions of the Galileo Test User Receiver.
- European Commission (EC) and Galileo Joint Undertaking (GJU) sponsorship of industrial receiver capabilities. These programs include the GARDA and Safety of Life programs undertaken by Alcatel Alenia Space – Italia.
- Canadian Space Agency (CSA) sponsorship of NovAtel Galileo receiver development has resulted in the Galileo Test Receiver (GTR) for CSA independent signal testing. GTR development is also providing technology that NovAtel is using in the Galileo Reception Chain (GRC) receiver development for the Galileo ground control network.

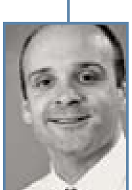
These publicly sponsored activities have led to some early commercial spin-offs that are providing users with the means to acquire and evaluate early Galileo signals.

Septentrio's GeNeRx1 is a combined GPS/Galileo receiver that can be configured to simultaneously track Galileo and GPS. Its 54 channels can be flexibly configured to simultaneously track up to 6 Galileo signals (L1, E5a, E5b, E5 (AltBOC) or E6) as well as up to 48 GPS L1 or L2 signals. GeNeRx1 supports both the Galileo Satellite Test Bed version 2 (GSTBv2) and the Galileo signal in space interface control document (SIS-ICD), and can also track the new GPS L5 signal.

Septentrio also recently announced its new AsteRx1 GPS/Galileo L1 GPS/Galileo/SBAS OEM receiver platform. Both GIOVE and Galileo modulations are supported on the 24-channel board with update rates of up to 50 Hz.

The NovAtel EuroPak-L1L5E5a is a configurable GPS/Galileo 16-channel, single-card receiver in a EuroPak enclosure. The receiver may be configured to receive a number of combinations of GPS L1 and L5 and Galileo L1 and E5a signals (GIOVE and SiS ICD), and is supplied with GPSolution GUI support. NovAtel has also released the GPS-704X wideband, passive antenna for use with this and other GPS/Galileo/GLONASS receivers. The L1L5E5a receiver is sold initially configured for GPS L1/L5 only, until ESA authorization is received, when Galileo functionality will be provided through no-charge field updates.

In addition to these sponsored



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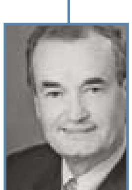

Professor Gérard Lachapelle holds a CRC/iCORE Chair in Wireless Location in the Department of Geomatics Engineering at the University of Calgary. He has been involved with GNSS since 1980 and has received numerous awards for his contributions in the area of differential kinematic GPS and indoor location.

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receiver developments, at least two GNSS manufacturers have advertised the availability of receivers for upgrade to handle future Galileo signals. Javad Navigation Systems (JNS) offers the GeNiuSS chip with 72 channels and fully configurable to track all current and future GPS, GLONASS, and Galileo signals. Topcon Positioning Systems, which also uses the JNS chip, has similarly advertised the same upgradeability of their receivers with G3 Paradigm capability.

Trimble Navigation has also indicated that it has plans for introducing Galileo capability on some of its receivers, and other manufacturers are well along in developing Galileo receivers in both software and hardware formats.

Meanwhile, receiver developers within the Galileo program are currently unable to supply Galileo capable receivers outside the program, as the Galileo SiS ICD is restricted. However, ESA has indicated that the ICD will shortly be provided for open release, as soon as initial frequency allocation testing of the GIOVE-A satellite is completed. Hopefully, this will allow scientific and agency users to access available receivers able to acquire the signal and track the GIOVE-A/B and IOV satellites that are to follow.

In summary, much receiver development is under way in Europe related to the infrastructure of the Galileo program. Some of this technology has already been spun off into early GPS/Galileo receivers in Europe. Similar activity in Canada has resulted in a commercially available receiver and antenna pair.

Some commercial vendors are advertising that existing receivers may be updated to add Galileo capability in the future. All receivers need both GPS and Galileo functionality to capture the many benefits from a large, combined GNSS constellation, and benefits will be seen gradually over time as the Galileo constellation grows.

TONY MURFIN

Tony Murfin, vice president for business development at NovAtel, Inc., based in Calgary, Alberta, Canada, is responsible for the aviation business group and other strategic business initiatives for the company. He has been responsible for the Wide Area Augmentation System receiver programs and is now involved in the European Galileo receiver program. Murfin has a B.Sc. from the University of Manchester (UK) Institute of Science and Technology and is a UK Chartered Engineer (CEng MIEE). IG